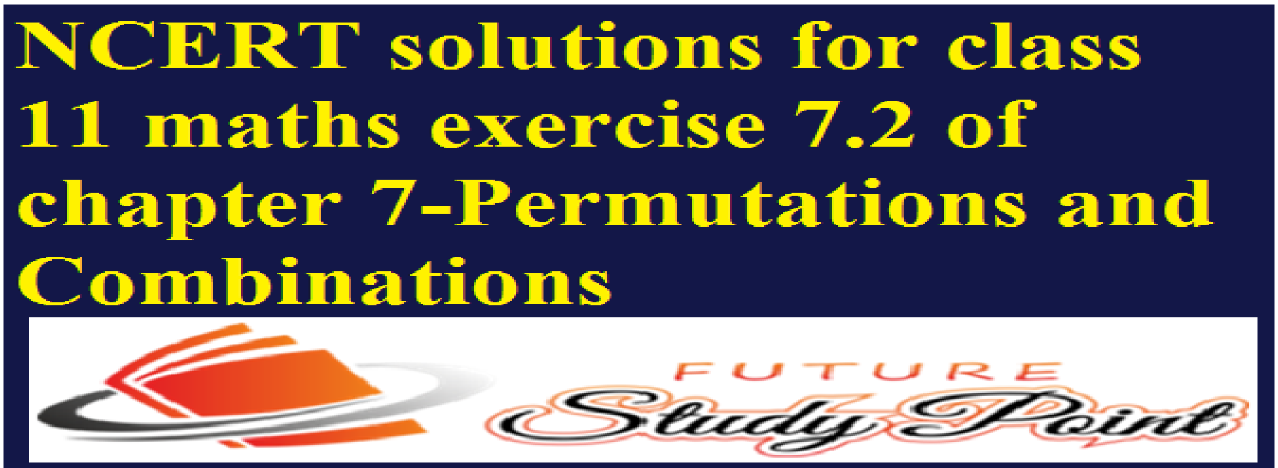
NCERT solutions for class 11 maths exercise 7.2 of chapter 7-Permutations and Combinations



NCERT solutions for class 11 maths exercise 7.2 of chapter 7-Permutations and Combinations are prepared for helping the students in their maths study so that they could clear their maths concepts.NCERT solutions for class 11 maths exercise 7.2 of chapter 7-Permutations and Combinations are based on permutations.A permutation is the number of arrangements formed by choosing a certain number of objects at a time from a set of objects.

Exercise 7.1- Permutations and Combinations

NCERT solutions of class 11 maths

CBSE Class 11-Question paper of maths 2015

CBSE Class 11 - Second unit test of maths 2021 with solutions

Study notes of Maths and Science NCERT and CBSE from class 9 to 12

Q1. Evaluate

(i) 8 !   (ii) 4 ! - 3 !

Ans.(i) $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

(i) $4! - 3! = 4 \times 3 \times 2 \times 1 - 3 \times 2 \times 1 = 24 - 6 = 18$

Q2. Is 3 ! + 4 ! = 7! ?

Ans. Taking LHS

$3! + 4! = 3 \times 2 \times 1 + 4 \times 3 \times 2 \times 1 = 6 + 24 = 30$

RHS= $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$\therefore 3! + 4! \neq 7!$

Q3. Compute :

$$\frac{8!}{6! \times 2!}$$

Ans.

$$\frac{8!}{6! \times 2!}$$

$$= \frac{8 \times 7 \times 6!}{6! \times 2 \times 1}$$

$$= \frac{56}{2} = 28$$

$$Q4. If\ \frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}, find\ x$$

Ans.

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

$$\frac{1}{6!} + \frac{1}{7 \times 6!} = \frac{x}{8 \times 7 \times 6!}$$

$$\frac{1}{6!}\left(1 + \frac{1}{7}\right) = \frac{x}{56 \times 6!}$$

$$\frac{8}{7} = \frac{x}{56}$$

x = 64

$Q5. Evaluate \ \frac{n!}{(n-r)!}, \ when$

(i) n = 6,r= 2 (ii) n = 9, r = 5

Ans.(i) n = 6,r= 2

(ii) n = 9, r = 5

Exercise 7.3

Q1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Ans. The given digits are 9, i.e 1,2,3....9

The number of 3 digits numbers using 9 digits = Permutations of 9 different digits choosing 3 digits at a time

Hence required number of 3 digit numbers

$= {}^9P_3$

$$= \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!} = 9 \times 8 \times 7 = 504$$

Q2. How many 4-digit numbers are there with no digit repeated?

Ans. The digits utilized to form 4 digits numbers are 0,1,2,3....9

Number of 4-digit numbers with no digit repeated = Number of permutations of 9 different digits choosing 4 digits at a time

Hence required number of 4 digit numbers

$= {}^{10}P_4$

$$\frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10 \times 9 \times 8 \times 7 \times \times 6!}{6!} = 5040$$

Since the number of 5040 also includes the numbers which starts from 0 i.e 0123,0324 etc,but actually these are three digits numbers

Therefore the number of permutations of 9 different digits (o is excluded) choosing 3 digits at a time

$= {}^{9}P_4$

$$\frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!} = 9 \times 8 \times 7 = 504$$

Therefore the required 4 digit numbers are = 5040 - 504 = 3536

Q3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6,7 if no digit is repeated.

Ans. Let's find out 3-digit even numbers using the digits 1, 2, 3, 4,6,7

For a three-digit even number , any of the 3 digits 2,4, or 6 is required in its unit place.

The number of ways of filling unit place is = The number of permutations of 3 different digits choosing 1 digits (2,4,or 6) at a time

$= {}^{3}P_1$

$$= \frac{3!}{(3-1)!} = \frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = \frac{6}{2} = 3$$

Since digits are not allowed to be repeated then the number of ways of filling tens and hundred places are = Number of permutations of remaining 5 digits choosing 2 digits at a time

$= {}^{5}P_2$

$$= \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times 3!}{3!} = 20$$

According to the multiplication principle the number of ways creating three-digit even numbers are = 3 × 20 = 60

Therefore required 3 digit even numbers are = 60

Q4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

Ans.The number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated = Number of permutations of 5 digits choosing 4 digits at a time

$= {}^5P_4$

$$= \frac{5!}{(5-4)!} = \frac{5!}{1!} = \frac{5 \times 4 \times 3 \times 2 \times 1!}{1!} = 120$$

Let's find out 4-digit even numbers using the digits 1, 2, 3, 4,5

For afour-digit even number , any of the 2 digits 2,4 is required in its unit place.

The number of ways of filling unit place is = The number of permutations of 2 different digits choosing 1 digits (2,4,) at a time

$= {}^2P_1$

$$= \frac{2!}{(2-1)!} = \frac{2!}{1!} = \frac{2 \times 1}{1} = 2$$

Since digits are not allowed to be repeated then the number of ways of filling tens and hundred and thousand places are = Number of permutations of remaining 4 digits choosing 3 digits at a time

$= {}^4P_3$

$$= \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4 \times 3 \times 2}{1} = 24$$

According to the multiplication principle the number of ways of creating four-digit even numbers are = 2 × 24 =48

Therefore required 4 digit even numbers are 48 and number of 4-digit numbers that can be formed are 120.

Q5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice-chairman assuming one person can not hold more than one position?

Ans. From a committee of 8 persons, let's find out the number of ways of choosing a chairman and a vice-chairman assuming one person can not hold more than one position

The number of ways of choosing a chairman and a vice-chairman out of a committee of 8 persons = The number of permutations of 8 different persons choosing 2 persons at a time

$= {}^{8}P_{2}$

Q6. Find n if ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$

Ans. ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$

$$\frac{(n-1)!}{(n-4)!} : \frac{n!}{(n-4)!} = \frac{1}{9}$$

$$\frac{(n-1)!}{n!} = \frac{1}{9}$$

$$\frac{(n-1)!}{n\,(n-1)!} = \frac{1}{9}$$

$$\frac{1}{n} = \frac{1}{9}$$

n = 9

Q7. Find r if (i) ${}^{5}P_r = 2.{}^{6}P_{r-1}$ (ii) ${}^{5}P_r = {}^{6}P_{r-1}$

Ans.

(i) ${}^{5}P_r = 2.{}^{6}P_{r-1}$

$$\frac{5!}{(5-r)!} = 2.\frac{6!}{(6-r+1)!}$$

$$\frac{5!}{(5-r)!} = 2.\frac{6 \times 5!}{(7-r)!}$$

$$\frac{12}{(7-r)(6-r)} = 1$$

(7 -r)(6 -r) = 12

42 - 7r - 6r + r² = 12

$r^2 - 13r + 30 = 0$

$r^2 - 10r - 3r + 30 = 0$

r(r- 10) - 3(r - 10) =0

(r- 10)(r - 3) = 0

r = 10,3

In ^nP_r ,0≤r ≤n, therefore the value of r is 3

(ii) $^5P_r = {}^6P_{r-1}$

$$\frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$$

$$\frac{6}{(7-r)(6-r)} = 1$$

(7 - r)(6 - r) = 6

$42 - 7r - 6r + r^2 = 6$

$r^2 - 13r + 36 = 0$

$r^2 - 9r - 4r - 36 = 0$

r(r - 9) - 4(r - 9) = 0

(r - 9) (r - 4) = 0

r =9,4

In ^nP_r ,0≤r ≤n, therefore the value of r is 4

Q8. How many words can be, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter once?

Ans. The number of letters in the word EQUATION are = 8

The number of ways,the words with or without meaning, can be formed using all 8 letters = Number of permutations of 8 letters choosing 8 letters at a time

$= {}^{8}P_{8}$

$$\frac{8!}{(8-8)!} = \frac{8!}{0!} = 8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 40320$$

Q9. How many words can be, with or without meaning, can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.

(i) 4 letters are used at a time

(ii) all letters are used at a time

(iii) all letters are used but first letter is vowel?

Ans.(i) The number of letters in the word MONDAY are = 6

The number of ways,the words with or without meaning, can be formed using 4 letters at a time out of 6 letters= Number of permutations of 6 letters choosing 4 letters at a time

$= {}^{6}P_{4}$

(ii) The number of ways,the words with or without meaning, can be formed using all 6 letters at a time out of 6 letters= Number of permutations of 6 letters choosing 6 letters at a time

$= {}^{6}P_{6}$

(iii) The number of vowels in the word MONDAY is = 2 (i.e O, A)

The number of ways the words formed using 1 vowel at a first place out of 2 vowels= Number of permutations of 2 vowels choosing 1 vowel at a time

$= {}^{2}P_{1} = 2$

Since one place is already occupied by one vowel therefore the number of ways,of selecting the remaining 5 letters at a time = Number of permutations of 5 letters choosing 5 letters at a time

= ${}^5P_5$ = 5! = 5×4×3×2= 120

According to the multiplication principle, the number of ways of creating 6 letters words are = 2 × 120 =240

Q10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Ans. The total number of words in MISSISSIPPI are = 11

The frequency of I = 4

The frequency of S = 4

The frequency of P = 2

We have to determine

Distinct permutations of the letters in word when 4I's not come together= Distinct permutations of the letters in the word -Distinct permutations of the letters in word when 4I's come together

Number of words formed by using all the letters of the word =The distinct permutations of the letters in the word

When 4I's come together should be treated as a single object, therefore in this case total letters would be supposed as 8, with the frequency of S =4 and P=2

The distinct permutations of the letters in the word when 4 I's are together

$$= \frac{8!}{4!2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{4 \times 3 \times 2 \times 2} = 840$$

Distinct permutations of the letters in word when 4I's not come together =34650-840= 33810

Q11. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) words start with P and end with S

(ii) vowels are all together

(iii) there are always 4 letters between P and S?

Ans. (i) The number of letters in word PERMUTATIONS are = 12

Let's find out the words start with P and end with S, means first and last letter are fixed, permutations of letters of the remaining 10 letters in which there are 2 T 's

Therefore in this case the number of ways of arrangement

$$= \frac{10!}{2!} = \frac{10 \times 9 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{2} = 1814400$$

(ii) In this case when vowels are all together,so all vowels together are treated as single letter.There are 5 vowels A,E,I,O,U, reamaining 7 letters and 5 vowels treated as 8 letters with 2 T's

Therefore in this case the number of ways of arrangement

$$= \frac{8!}{2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{2} = 20160$$

The number of ways of arrangement of 5 vowels = 5! = 5×4×3×2=120

Hence according to multiplication principle total number of arrangements in this case =20160 × 120 = 2419200

(iii) There are always 4 letters between P and S, it can be shown as below.

$P - - - - S - - - - - -$

$-P - - - - S - - - - -$

$- - P - - - - S - - - -$

$- - - P - - - - S - - -$

$- - - - P - - - - S - -$

$- - - - - P - - - - S -$

$- - - - - - P - - - - S$

It can be observed that the number of ways of placing 4 letters between P and S are = 7 ways

The way of arrangement of P and S is =2! = 2

The remaining 10 letters can be arranged with 2 I's by (10!)/2!

According to the multiplication principle, the number of ways of placing 4 letters between P and S   are = 7×2 × (10!)/2! =25401600

Permutations

A permutation is the number of arrangements formed by choosing a certain number of objects at a time from a set of objects.Let there are n objects and we have to creat the arrangement of the objects with r objects taken at a time then permutations of the n objects r taken at a time

$$^nP_r = \frac{n!}{(n-r)!}$$

If certain objects repeated by m and t times then permutations of all objects taken at a time

$$= \frac{n!}{t!m!}$$

Permutations are the way of arrangement of the things when order also matters, which means in the arrangement if we have taken 23 then 32 is also counted.